Enrollment No: ______________________ **Exam Seat No:**______________________

C.U.SHAH UNIVERSITY

Summer Examination-2019

Subject Name : Engineering Mathematics - 4

Subject Code : 4TE04EMT2 **Branch: B.Tech (Civil/EE/Mech)**

Semester : 4 **Date : 15/04/2019** **Time : 02:30 To 05:30** **Marks : 70**

Instructions:

(1) Use of Programmable calculator & any other electronic instrument is prohibited.
(2) Instructions written on main answer book are strictly to be obeyed.
(3) Draw neat diagrams and figures (if necessary) at right places.
(4) Assume suitable data if needed.

Q-1 **Attempt the following questions:** **(14)**

a) δ equal to

(A) $\frac{\Delta}{E^{\frac{1}{2}}}$ (B) $E^{\frac{1}{2}}+E^{\frac{-1}{2}}$ (C) $E^{\frac{1}{2}}-E^{\frac{-1}{2}}$ (D) none of these

b) E equal to

(A) $1+\Delta$ (B) $\Delta\nabla$ (C) $\nabla+\Delta$ (D) $\nabla-\Delta$

c) Putting $n=2$ in the Newton – Cote's quadrature formula following rule is obtained

(A) Simpson's $\frac{1}{3}$ rule (B) Trapezoidal rule (C) Simpson's $\frac{3}{8}$ rule

(D) none of these

d) While evaluating a definite integral by Trapezoidal rule, the accuracy can be increased by taking

(A) small number of sub – intervals (B) large number of sub – intervals

(C) odd number of sub – intervals (D) none of these

e) The convergence in the Gauss – Seidel method is faster than Gauss – Jacobi method.

(A) True (B) False

f) The Gauss – Jordan method in which the set of equations are transformed into diagonal matrix form.

(A) True (B) False

g) Which of the following methods is the best for solving initial value problems:

(A) Taylor's series method (B) Euler's method

(C) Runge-Kutta method of 4^{th} order (D) Modified Euler's method

h) Using modified Euler's method, the value of $y(0.1)$ for $\frac{dy}{dx}=x-y,\ y(0)=1$ is

(A) 0.909 (B) 0.809 (C) 0.0809 (D) 0.0908

i) The finite Fourier cosine transform of $f(x)=2x,\ 0<x<4$ is

(A) $\frac{32}{n^2\pi^2}\left[(-1)^n - 1\right]$ (B) $\frac{16}{n^2\pi^2}\left[(-1)^n - 1\right]$ (C) $\frac{32}{n^2\pi^2}(-1)^n$ (D) none of these

j) The Fourier sine transform of $f(x)=\begin{cases}1, & 0<x<a\\ 0, & x>a\end{cases}$ is

(A) $\sqrt{\frac{2}{\pi}}\left(\frac{1+\cos a\lambda}{\lambda}\right)$ (B) $\sqrt{\frac{2}{\pi}}\left(\frac{1-\cos a\lambda}{\lambda^2}\right)$ (C) $\sqrt{\frac{2}{\pi}}\left(\frac{1-\cos a\lambda}{\lambda}\right)$

(D) none of these

k) If $w = \mathrm{f}(z) = \mathrm{u}(x,y) + i\mathrm{v}(x,y)$ is analytic then $f'(z)$ equal to

(A) $\frac{\partial u}{\partial x} - i\frac{\partial u}{\partial y}$ (B) $\frac{\partial u}{\partial x} - i\frac{\partial v}{\partial x}$ (C) $\frac{\partial v}{\partial y} - i\frac{\partial v}{\partial x}$ (D) none of these

l) The image of circle $|z-1| = 1$ in the complex plane, under the mapping $w = \frac{1}{z}$ is

(A) $|w-1| = 1$ (B) $u^2 + v^2 = 1$ (C) $v = \frac{1}{z}$ (D) $u = \frac{1}{z}$

m) If $\phi = xyz$, the value of $|\text{grad }\phi|$ at the point $(1,2,-1)$ is

(A) 0 (B) 1 (C) 2 (D) 3

n) If $\vec{A}(t) = 3t^2 i + 4tj + 4t^3 k$, $\int_{t=1}^{t=2} \vec{A}(t)\, dt$ equal to

(A) $15i + 6j + 7k$ (B) $7i + 6j + 5k$ (C) $7i + 15j + 6k$ (D) none of these

Attempt any four questions from Q-2 to Q-8

Q-2 **Attempt all questions** **(14)**

a) Consider following tabular values **(5)**

x	50	100	150	200	250
y	618	724	805	906	1032

Using Newton's Backward difference interpolation formula determine $y(300)$.

b) Use Stirling's formula to find y_{28} given **(5)**

that $y_{20} = 49225,\ y_{25} = 48316,\ y_{30} = 47236,\ y_{35} = 45926$ and $y_{40} = 44306$.

c) Find the finite Fourier cosine transform of $f(x) = 2x,\quad 0 < x < 4$. **(4)**

Q-3 **Attempt all questions** **(14)**

a) Solve the following system of equations using Gauss-Seidel Method: **(5)**

$30x - 2y + 3z = 75,\ 2x + 2y + 18z = 30,\ x + 17y - 2z = 48$

b) Given that **(5)**

x	1.00	1.05	1.10	1.15	1.20	1.25	1.30
y	1.00000	1.02470	1.04881	1.07238	1.09544	1.11803	1.14017

Find $\frac{dy}{dx}$ at $x = 1.05$.

c) If $f(z) = u + iv$ is an analytic function of z and $u + v = e^x(\cos y + \sin y)$ then find $f(z)$. **(4)**

Q-4 **Attempt all questions** **(14)**

a) Apply Runge-Kutta fourth order method, to find an approximate value of y when **(5)**

$x = 0.2$, given that $\frac{dy}{dx} = x + y$ and $y = 1$ when $x = 0$.

b) Evaluate $\int_0^6 \frac{1}{1+x^2}\, dx$ by using Simpson's 3/8th rule. **(5)**

c) Solve the following system of equations using Gauss-Jordan method: **(4)**
$x + 2y + z = 3,\ 2x + 3y + 3z = 10,\ 3x - y + 2z = 13$

Q-5 **Attempt all questions** **(14)**

a) Using Cauchy's integral formula, evaluate $\oint_C \frac{z^4}{(z+1)(z-i)^2}\, dz$, where C is the ellipse $9x^2 + 4y^2 = 36$. **(5)**

b) If $\vec{F} = (2x^2 - 4z)i - 2xyj - 8x^2k$, then evaluate $\iiint_V div\ \vec{F}\ dV$, where V is bounded by the planes $x = 0$, $y = 0$, $z = 0$, $x + y + z = 1$. **(5)**

c) The following table gives the values of x and y: **(4)**

x	30	35	40	45	50
y	15.9	14.9	14.1	13.3	12.5

Use Lagrange's inverse interpolation formula to find the value of x corresponding to $y = 13.6$.

Q-6 **Attempt all questions** **(14)**

a) Prove that $\vec{F} = (y^2 \cos x + z^3)i + (2y \sin x - 4)j + 3xz^2k$ is irrotational and find its scalar potential. **(5)**

b) Under the transformation $w = \frac{1}{z}$ **(5)**

(a) Find the image of $|z - 2i| = 2$

(b) Show that the image of the hyperbola $x^2 - y^2 = 1$ is the lemniscates $\rho^2 = \cos 2\theta$.

c) Obtain Picard's second approximation solution of the initial value problem $\frac{dy}{dx} = x^2 + y^2$ for $x = 0.4$ correct to four decimal places, given that $y(0) = 0$. **(4)**

Q-7 **Attempt all questions** **(14)**

a) Show that the function $f(z) = \sqrt{|xy|}$ is not analytic at the origin, although Cauchy-Riemann equations are satisfied. **(5)**

b) Using Green's Theorem, evaluate $\int_C (3x^2 - 8y^2)dx + (4y - 6xy)dy$ where C is the boundary of the region bounded by $y^2 = x$ and $y = x^2$. **(5)**

c) Evaluate $\int_0^1 x^3\, dx$ by Trapezoidal Rule using 5 subintervals. **(4)**

Q-8 **Attempt all questions** **(14)**

a) Solve $\frac{dy}{dx} = x + y$ with $y(0) = 1$ by Euler's modified method for $x = 0.1$ correct to four decimal places by taking $h = 0.05$. **(5)**

b) Using Fourier integral show that $\int_0^\infty \frac{1-\cos \pi\lambda}{\lambda} \sin x\lambda \, d\lambda = \begin{cases} \frac{\pi}{2} & \text{if } 0 < x < \pi \\ 0 & \text{if } x > \pi \end{cases}$ **(5)**

c) Prove that the angle between the surface $x^2 + y^2 + z^2 = 9$ and $x^2 + y^2 - z = 3$ at the point $(2, -1, 2)$ is $\cos^{-1}\left(\frac{8}{3\sqrt{21}}\right)$. **(4)**